MENIIT
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JEE Advanced : Paper-1 (2013)

IMPORTANT INSTRUCTIONS

A. General:

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are NOT allowed inside the examination hall.
3. Write your name and roll number in the space provided on the back cover of this booklet.
4. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. These parts should only be separated at the end of the examination when instructed by the invigilator. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be retained by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
5. **Using a black ball point pen darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom duplicate sheet.

B. Question Paper Format :

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

Section 1 constrains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

Section 2 contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

Section 3 contains **5 questions**. The answer to each question is a single–digit integer, ranging from 0 to 9 (both inclusive)

C. Marking Scheme:

For each question in **Section 1**, you will be awarded **2 marks** if your darken the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **4 marks** if you darken all the bubble(s) corresponding to only the correct answer(s) and zero mark if no bubbles are darkened. In all other cases, minus one (–1) mark will be awarded.

For each question in **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (–1) mark will be awarded.

PART-A : PHYSICS

Section - 1 :

(Only One option correct Type)

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct. **(10 Q × 2 M = No Negative)**

1. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n+1)\frac{\lambda}{2}$ (B*) $(2n+1)\frac{\lambda}{4}$ (C) $(2n+1)\frac{\lambda}{8}$ (D) $(2n+1)\frac{\lambda}{16}$

Ans. **[B]**

Sol. $\frac{I_0}{2} = I_R = I_0 \cos^2\left(\frac{\pi}{\lambda}\Delta P\right)$

$\Rightarrow \quad \cos\left(\frac{\pi}{\lambda}\Delta P\right) = \pm\frac{1}{\sqrt{2}}$

$\therefore \Delta P = (2n+1)\frac{\lambda}{4}$

2. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is

(A) 0.3×10^{-17} kg ms^{-1} (B*) 1.0×10^{-17} kg ms^{-1}

(C) 3.0×10^{-17} kg ms^{-1} (D) 9.0×10^{-17} kg ms^{-1}

Ans. **[B]**

Sol. $P = \frac{hc}{\lambda c} = mV = \frac{Et}{ct} = \frac{pt}{c}$

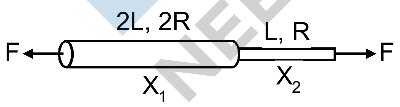
$= \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}} = 10^{-17}$

3. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is strectched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50 (C*) 2.00 (D) 4.00

Ans. **[C]**

Sol.



$K_1 = \frac{YA}{L}$ $K_2 = \frac{Y\pi R^2}{L} = K$

$= \frac{Y4\pi R^2}{2L} = 2K$

$2Kx_1 = kx_2$

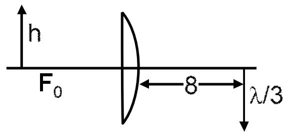
$\frac{x_2}{x_1} = 2$

4. The image of an object, formed by a plano-convex lens at distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

(A) 1 m (B) 2 m (C*) 3 m (D) 6 m

Ans. **[C]**

Sol. $M = \frac{-x_2}{f}$



$\Rightarrow \frac{-1}{3} = \frac{-8}{f}$ $\therefore$ f = 24 cm

$\frac{1}{f} - (\mu - 1)\frac{1}{R}$

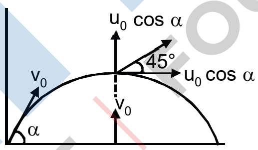
$\frac{1}{24} = \left(\frac{3}{2} - 1\right)\frac{1}{R}$ $\therefore$ R = 12 cm

5. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(A*) $\frac{\pi}{4}$ (B) $\frac{\pi}{4} + \alpha$ (C) $\frac{\pi}{2} - \alpha$ (D) $\frac{\pi}{2}$

Ans. A

Sol. $H = \frac{u_0^2 \sin^2 \alpha}{2g}$



$v^2 - u_0^2 = \frac{2(-g)u_0^2 \sin^2 \alpha}{2g}$

$v^2 = u_0^2 \cos^2 \alpha$

6. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0), to the point (0, a) along a circular path of radius a about the origin in the x-y plane is

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D*) 0

Sol. $\vec{F} = \left[\frac{x\hat{i} + y\hat{j}}{(x^2+y^2)^{3/2}}\right]$

is perpendicular to the displacement

$\therefore$ WD = 0

7. The diameter of a cylinder is measured using a Vernier Callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B*) 5.124 cm (C) 5.136 cm (D) 5.148 cm

Ans. **[B]**

Sol. Reading = 5.100 + 0.001 × 24 = 5.124

8. Two non-reactive monoatomic ideal gasses have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

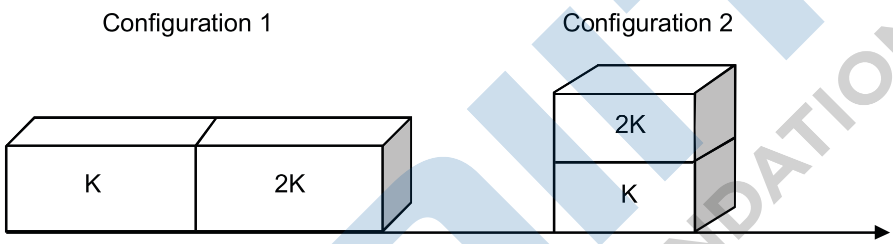
(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D*) 8 : 9

Ans. **[D]**

Sol. PM = ρRT

$$\frac{s_1}{s_2} = \frac{P_1}{P_2}\frac{M_1}{M_2} = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$$

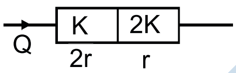
9. Two rectangular, blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and the other 2K. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is



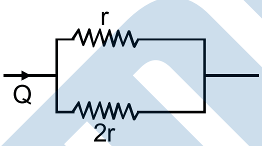
(A*) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

Ans. **[A]**

Sol.



$\Delta T = \frac{Q}{t_1}(3r)$ .... (1)



$\Delta T = \frac{Q}{t_2}\frac{2r}{3}$ .... (2)

$\Rightarrow \frac{t_1}{t_2} = \frac{3r}{2r}\times 3$

$t_2 = \frac{2}{9} \times 9 = 2s$

10. A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$. The angle of incidence is

(A*) 30° (B) 45° (C) 60° (D) 75°

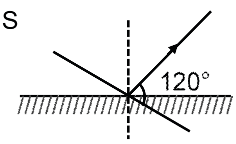
Ans. **[A]**

Sol. $\cos\theta = \dfrac{\vec{a}.\vec{b}}{ab}$

$= \dfrac{1/4(1-3)}{1} = \dfrac{1}{2}$

$\therefore\ \theta = 120°$

$\therefore\ \alpha = 30°$



Section - 2 :
(One or more options correct Type)

This section contains **5 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE** may be correct. **(5 Q × 4 M = 20 and – 1 negative)**

11. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A*) the net elongation of the spring is $\dfrac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\dfrac{8\pi R^3 \rho g}{3k}$

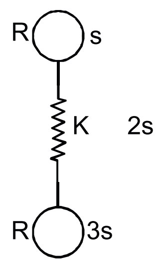
(C) the light sphere is partially submerged

(D*) the light sphere is completely submerged.

Ans. AD

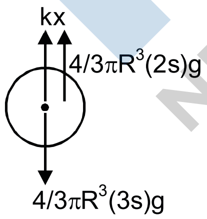
Sol. If completely submerged

$F_B = \left(\dfrac{4}{3}\pi R^3\right) \times 2 \times 2s = \dfrac{16}{3}\pi R^3 s$



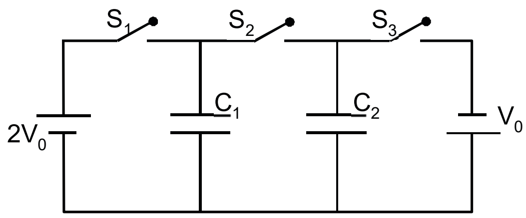
Total mass = $\dfrac{4}{3}\pi R^3\, 3s + \dfrac{4}{3}\pi R^3\, s$

$= \dfrac{16}{3}\pi R^3 s$



$\therefore\ kx = 4/3\pi R^3\ sg$

12. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time,



(A) the charge on the upper plate of C_1 is $2CV_0$

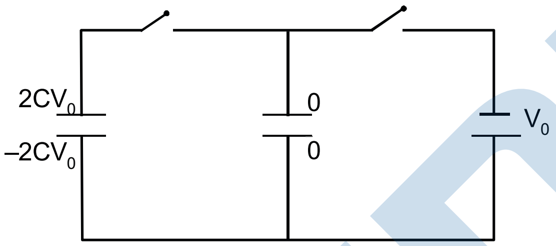
(B*) the charge on the upper plate of C_1 is CV_0

(C) the charge on the upper plate of C_2 is 0.

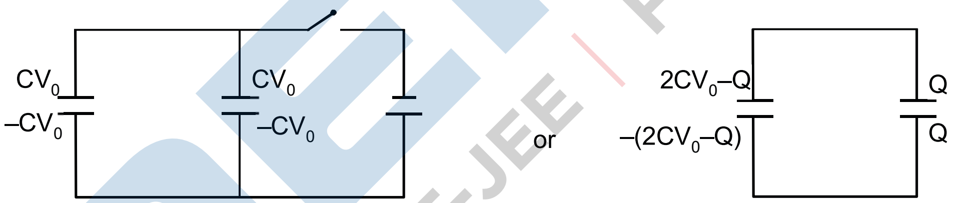
(D*) the charge on the upper plate of C_2 is $-CV_0$

Ans. **[BD]**

Sol. Initial step

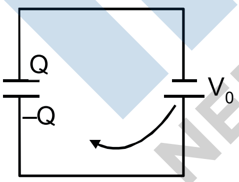


2^{nd} step

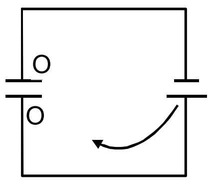


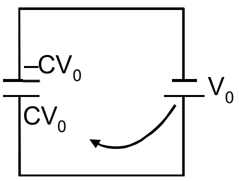
can be partial charge

3^{rd} step



Intermediate time



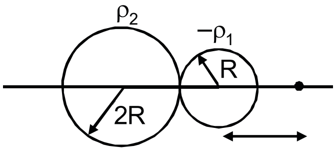


after a long time, charge will go from +Q to $-CV_0$ can be B, C, D

13. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(A) –4 (B*) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D*) 4

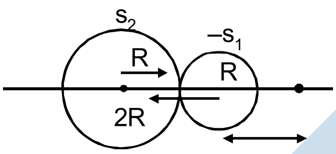
Ans. **[BD]**



Sol. (D)

$$\frac{K\left(\rho_1 \frac{4}{3}\pi R^3\right)}{(2R)^2} = \frac{K\left[\rho_2 \frac{4}{3}\pi (2R)^2\right]}{(5R)^2}$$

$$\frac{\rho_1}{\rho_2} = \frac{8 \times 4}{25}$$



(D)

$$\frac{\rho_2 R}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} \frac{\left(\rho_1 \frac{4}{3}\pi R^3\right)}{(2R)^2}$$

$$\frac{\rho_1}{\rho_2} = 4$$

14. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, y(x, t) = (0.01 m) sin [(62.8 m^{-1}) x] cos [(628 s^{-1})t]. Assuming π = 3.14, the correct statement(s) is (are)

(A) The number of nodes is 5

(B*) The length of the string is 0.25 m

(C*) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m

(D) The fundamental frequency is 100 Hz

Ans. **[BC]**

Sol. (A) 6

(B) $\frac{5\lambda}{2} = L$

$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K}$ = 0.1 m

$\therefore L = \frac{5\lambda}{2}$

$= 5 \times 0.1 \frac{1}{2} = 0.25$ m

(C) antinode is at mid point

$\therefore$ 0.01 m

(D) $w = 628 = 2\pi f_5$

$f_5 = 40$ Hz

$\therefore$ fundamental = 8 Hz

15. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all value of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with velocity $\vec{u}_2 = 2\left(\sqrt{3}\,\hat{i} + \hat{j}\right) ms^{-1}$. The correct statement(s) is (are)

(A*) The direction of the magnetic field is –z direction

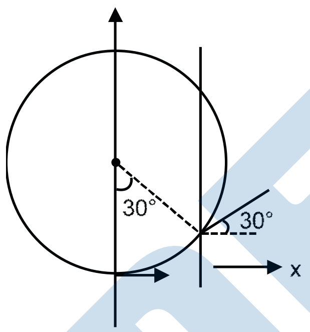
(B) The direction of the magnetic field is +z direction

(C*) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

Ans. [AC]

Sol.



$30^\circ \rightarrow 10$ ms

$360^\circ \rightarrow 120$ ms

$120 \times 10^{-3} = \frac{2\pi M}{QB}$

$B = \frac{\pi M}{QB} \times \frac{10^3}{60}$

Section - 3 :

(Integer value correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive). **(5 Q × 4 M = 20 and – 1 negative)**

16. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\ell n2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

Ans. **[4]**

Sol. $N_t = N_0\, e^{-\lambda t}$

$$\frac{N_0 - N_t}{N_0} = (1 - e^{-\lambda t})$$

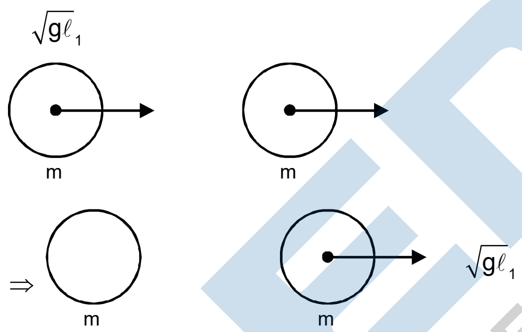
$$= \lambda t = \frac{\ell n2}{1386} \times 80$$

$= 4$

17. A bob of mass m, suspended by a string of length ℓ_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is

Ans. 5

Sol.

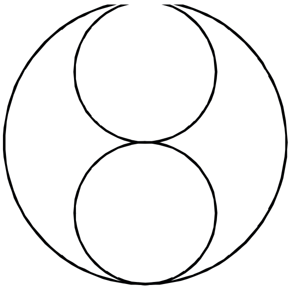


$\therefore \sqrt{g\ell_1} = \sqrt{5g\ell_2}$

$\therefore \frac{\ell_1}{\ell_2} = 5$

18. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc is such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

Ans. **[8]**

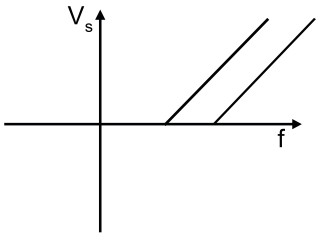
Sol. 

$$\frac{1}{2}MR^2\omega = \left[\frac{1}{2}MR^2 + 4mr^2\right]\omega_0$$

$$\omega_0 = \frac{MR^2\omega}{MR^2 + 8mr^2} = 8 rad/s$$

19. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

Ans. **[1]**

Sol. 

slope is same $\frac{h}{e}$

20. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is

Ans. 5

Sol. $\frac{1}{2}mv^2 = Pt$

$v = \sqrt{\frac{2Pt}{m}} = 5$ m/s

PART - B : CHEMISTRY

Section – 1

(Only one option correct Type)

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. Methylene blue, from its aqueous solution, is adsorbed on activated charcoal at 25°C. For this process, the correct statement is :

(A) The adsorption requires activation at 25°C.

(B*) The adsorption is accompanied by a decrease in enthalpy

(C) The adsorption increases with increase of temperature

(D) The adsorption is irreversible.

Ans **[B]**

Sol. Adsorption is physisorption and hence activation is almost not required. Physisorption is exothermic and reversible and it decreases with increase in temperature.

22. Upon treatment with ammoniacal H_2S, the metal ion that precipitates as a sulfide is

(A) Fe(III) (B) Al(III) (C) Mg(II) (D*) Zn(II)

Ans. **[D]**

Sol. II^{nd} group and IV^{th} group metals ions are ppt in form of sulphides.

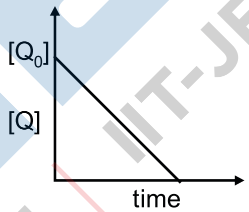
Fe (III) and Al (III) $\rightarrow$ III^{rd} group

Zn (II) $\rightarrow$ IV^{th} group

Mg (II) $\rightarrow$ V^{th} group

23. In the reaction,

$$P + Q \longrightarrow R + S$$



the time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is :

(A) 2 (B) 3 (C) 0 (D*) 1

Ans. **[D]**

Sol. $P + Q \longrightarrow R + S$

Given : $T_{75\%} = 2\, t_{50\%}$ for P

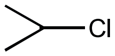
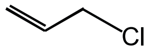
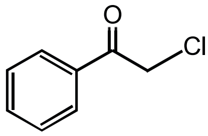
$\Rightarrow$ Reaction is of first order w.r.t. P

$\Rightarrow$ From graph it is clear that reaction is of zero order w.r.t. Q

$$[Q]_t = [Q]_0 - kt$$

so overall order will be = 1 + 0 = 1 Ans.

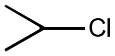
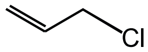
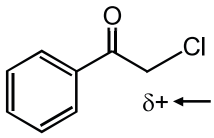
24. KI in acetone, undergoes S_N2 reaction with each of P, Q, R and S. The rates of the reaction vary as

$H_3C–Cl$   

P **Q** **R** **S**

(A) P > Q > R > S (B*) S > P > R > Q (C) P > R > Q > S (D) R > P > S > Q

Ans. **[B]**

Sol. $H_3C–Cl$   

δ+ ⟵ more charge

EWG & $-\underset{\underset{O}{\|}}{C}-$ group stabilize transition state also

P **Q** **R** **S**

Rate of SN_2 $\alpha \dfrac{1}{\text{steric hendrance}}$

α δ+ charge on carbon

α Stability of transition state

So, S > P > R > Q **Ans.**

25. The standard enthalpies of formation of $CO_2(g)$, $H_2O(l)$ and glucose(s) at 25°C are – 400 kJ/mol, – 300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25°C is :

(A) + 2900 kJ (B) –2900 kJ (C*) – 16.11 kJ (D) + 16.11 kJ

Ans. **[C]**

Sol. $C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O\ (\ell)$

$$\Delta H^\circ_{comb.C_6H_{12}O_6} = 6\Delta H^\circ_{f\,CO_2} + 6\Delta H^\circ_{f\,H_2O(\ell)} - \Delta H^\circ_{f\,C_6H_{12}O_6(s)}$$

$= 6 \times (-400) + 6 \times (-300) - (-1300)$

$= -2900$ kJ/mol

$\therefore$ Standard enthalpy of combustion per gram of glucose = $\dfrac{-2900}{180} = -16.11$ kJ

26. Consider the following complex ions, P, Q and R.

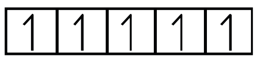
$P = [FeF_6]^{3-}$, $Q = [V(H_2O)_6]^{2+}$ and $R = [Fe(H_2O)_6]^{2+}$,

the correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is:

(A) R < Q < P (B*) Q < R < P (C) R < P < Q (D) Q < P < R

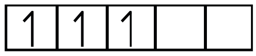
Ans. **[B]**

Sol. (P) $[FeF_6]^{-3} \rightarrow Fe^{+3} \rightarrow d^5$ configuration with WFL



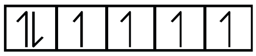
unpaired electron = 5

(Q) $[V(H_2O)_6]^{2+} \rightarrow V^{+2} \rightarrow d^3$ configuration



unpaired electron = 3

(R) $[Fe(H_2O)_6]^{2+} \rightarrow Fe^{+2} \rightarrow d^6$ configuration with WFL



unpaired electron = 4

Spin only magnetic moment (μ) = $\sqrt{n(n+2)}$ B.M. (n = unpaired electron)

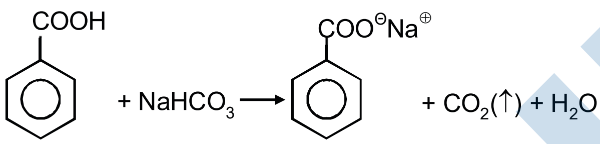
So, μ of P > R > Q

27. The compound that does **NOT** liberate CO_2 , on treatment with aqueous sodium bicarbonate solution, is:

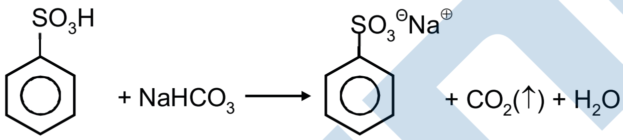
(A) Benzoic acid
(B) Bezenesulphonic acid
(C) Salicylic acid
(D*) Carbolic acid (Phenol)

Ans. **[D]**

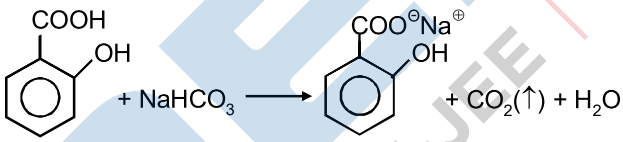
Sol.



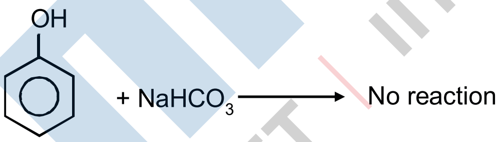
Benzoic acid



Benzene Sulphonic acid



Salicylic acid



Phenol

28. Sulfide ores are common for the metals :

(A*) Ag, Cu and Pb
(B) Ag, Cu and Sn
(C) Ag, Mg and Pb
(D) Al, Cu and Pb

Ans. **[A]**

Sol. Metals having high polarising power (Z_{eff}) exist in ore with anions having high polarisibility

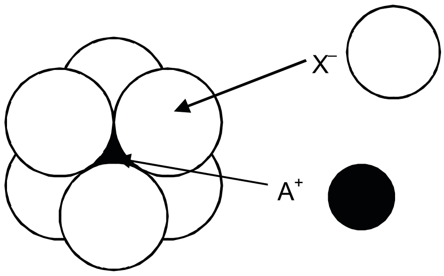
Ag, Cu, Pb $\rightarrow$ Sulphide ores

Sn $\rightarrow$ Oxide ore

Mg $\rightarrow$ Sea water

Al → Oxide ore

29. The arrangement of X^- ions around A^+ ion in solid AX is given in the figure (not drawn to scale). If the radius of X^- is 250 pm, the radius of A^+ is



(A*) 104 pm (B) 125 pm (C) 183 pm (D) 57 pm

Ans. **[A]**

Sol. It is an octahedral void

$$\frac{r_{A^+}}{r_{X^-}} = 0.414$$

$r_{A^+} = 0.414 \times 250 = 104$ pm **Ans.**

30. Concentrated nitric acid, upon long standing, turns yellow-brown due to the formation of :

(A) NO (B*) NO_2 (C) N_2O (D) N_2O_4

Ans. **[B]**

Sol. On standing

$$HNO_3 \rightarrow H_2O + NO_2 + 1/2\ O_2$$

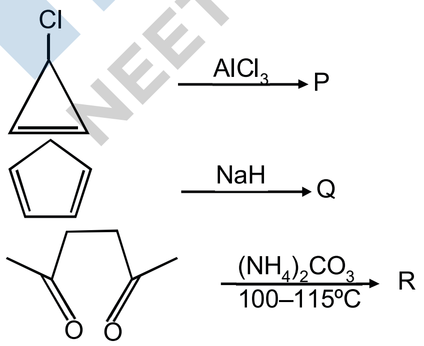
colour of NO_2 → yellow - brown

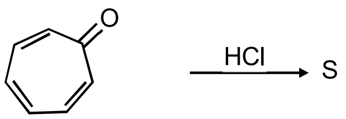
Section - 2 :

(One or more options correct type)

This section contains **5 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

31.$_{cc}$ Among P, Q, R and S, the aromatic compound(s) is / are

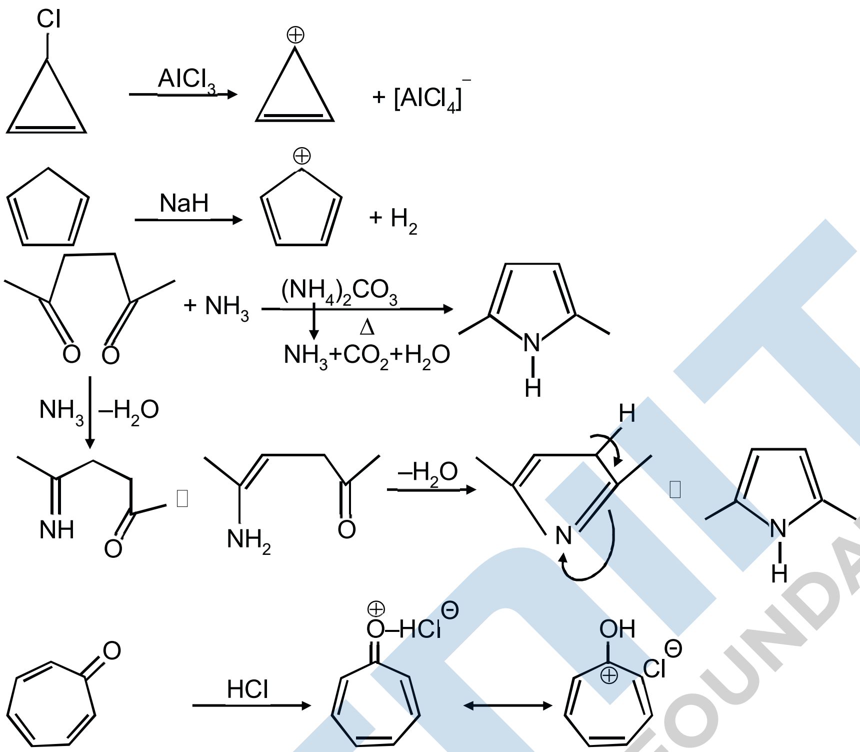




(A) P (B) Q (C) R (D) S

Ans. [ABCD]

Sol.



32. The initial rate of hydrolysis of methyl acetate (1M) by a weak acid (HA, 1M) is 1/100th of that of a strong acid (HX, 1M), at 25°C. The K_a of HA is

(A*) 1×10^{-4} (B) 1×10^{-5} (C) 1×10^{-6} (D) 1×10^{-3}

Ans. [A]

Sol. Rate = K [Ester] $[H^+]$

For strong acid $[H^+]$ = 1M

For weak acid $[H^+] = \sqrt{K_a.C}$

$$\frac{Rate_1}{Rate_2} = \frac{[H^+]_1}{[H^+]_2}$$

$$\frac{1}{1/100} = \frac{1}{\sqrt{K_a \times 1}}$$

$K_a = 10^{-4}$ M

33. Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement(s) is (are)

(A) ΔG is positive (B*) ΔS_{system} is positive (C*) $\Delta S_{surroundings} = 0$ (D*) $\Delta H = 0$

Ans. [BCD]

Sol. For ideal solution

$\Delta H_{solution} = 0$

$\Delta S_{solution} = +ve$

$\therefore\ \Delta G_{solution} = -\ ve$

$\Delta S_{surrounding} = 0$

(As no heat exchange takes place)

34. The pair(s) of coordination complexes/ ions exhibiting the same kind of isomerism is(are)

(A) $[Cr(NH_3)_5Cl]Cl_2$ and $[Cr(NH_3)_4Cl_2]Cl$

(B*) $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$

(C) $[CoBr_2Cl_2]^{2-}$ and $[PtBr_2Cl_2]^{2-}$

(D*) $[Pt(NH_3)_3(NO_3)]$ Cl and $[Pt(NH_3)_3Cl]$ Br

Ans. **[BD]**

Sol. (A) $[Cr(NH_3)_5Cl]Cl_2$ does not show any isomerism

$[Cr(NH_3)_4Cl_2]Cl$ show GI.

(B) $[Co(NH_3)_4Cl_2]^+$ show GI

$[Pt(NH_3)_2(H_2O)Cl]^+$ show GI

(C) $[CoBr_2Cl_2]^{2-}$ sp^3 hybridised does not show any isomerism

$[PtBr_2Cl_2]^{2-}$ dsp^2 hybridised show GI

(D) $[Pt(NH_3)_3(NO_3)]$ Cl show ionisation isomerism

$[Pt(NH_3)_3Cl]$ Br show ionisation isomerism

35. The hyperconjugative stabilities of tert-butyl cation and 2-butene, respectively, are due to

(A) $\sigma \rightarrow$ p (empty) and $\sigma \rightarrow \pi^*$ electron delocalistaions.

(B) $\sigma \rightarrow \sigma^*$ and $\sigma \rightarrow \pi$ electron delocalisations

(C) $\sigma \rightarrow$ p (filled) and $\sigma \rightarrow \pi$ electron delocalisations

(D) p (filled) $\rightarrow \sigma^*$ and $\sigma \rightarrow \pi^*$ electron delocalisations

Ans. **[A]**

Section - 3 :

(Integer value correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

36. A tetrapeptide has –COOH group on alanine. This produces glycine (Gly), Valine (Val), Phenyl Alanine (Phe) and Alanine (Ala), on complete hydrolysis. For this tetrapeptide, the number of possible sequences (primary structures) with $-NH_2$ group attached to a chiral center is

Ans. **[4]**

Sol. Tetrapeptide contains 3 peptide linkage and gives 4 amino acid on hydrolysis

Glycine (Gly) H_2N-CH_2-COOH

Alanine (Ala) $H_2N-CH(CH_3)-COOH$

Phenyl alanine (Phe) $H_2N-CH(CH_2-Ph)-COOH$

Valine (Val) $H_2N-CH(CH(CH_3)-CH_3)-COOH$

Since tetrapeptide contains free COOH at alanine so that alanine is last unit of tetrapeptide. Glycene does not contain chiral carbon so that Glycene cannot be at first position in tetrapeptide. Hence possible structure of this tetrapeptide will be 4

PVGA VGPA

PGVA VPGA

37. The atomic masses of He and Ne are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of He gas at –73°C is "M" times that of the de Broglie wavelength of Ne at 727°C. M is

Ans. **[5]**

Sol. $\lambda = \dfrac{h}{mV}$ $\quad\therefore\ \lambda \propto \dfrac{1}{\sqrt{TM}}$

$$\Rightarrow \frac{\lambda_{Ne}}{\lambda_{He}} = \frac{\sqrt{1200 \times 4}}{\sqrt{1000 \times 20}}$$

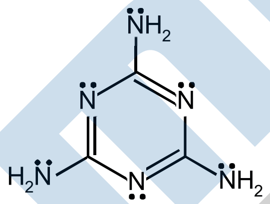
$$\Rightarrow \lambda_{He} = 5\,\lambda_{Ne}$$

$\therefore$ M = 5

38. The total number of lone-pairs of electrons in melamine is

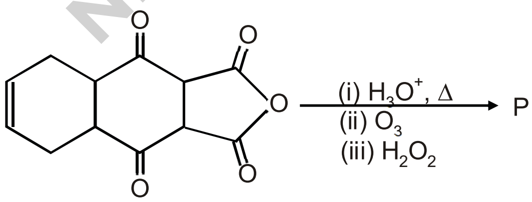
Ans. **[6]**

Sol.



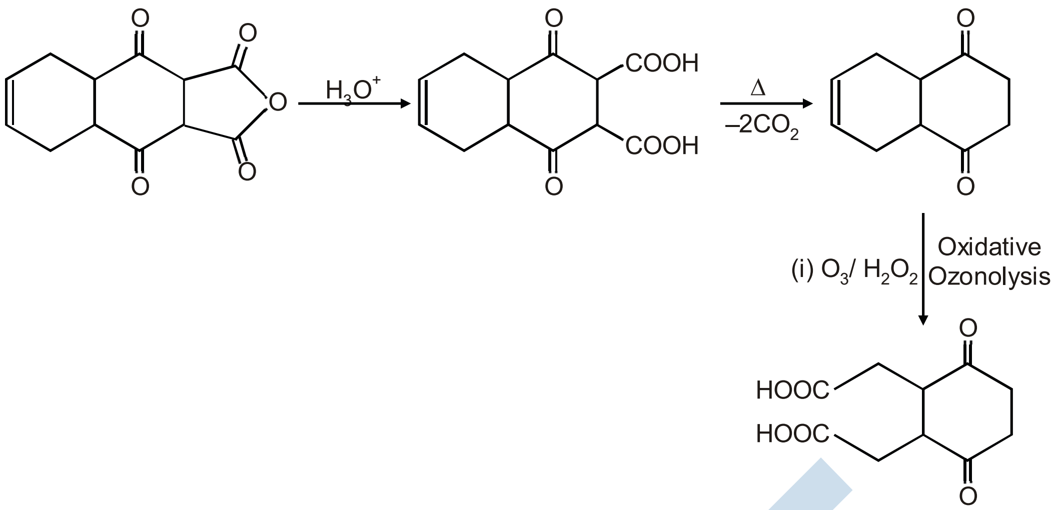
Number of lone pairs = 6

39. The total number of carboxylic acid groups in the product P is



Ans. **[2]**

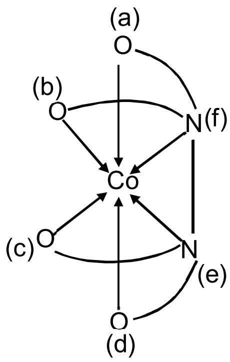
Sol.



40. EDTA^{4-} is ethylenediaminetetraacetate ion. The total number of N–Co–O bond angles in $[Co(EDTA)]^{1-}$ complex ion is

Ans. [8]

Sol.



N – Co – O bond angles

$a\hat{g}f$, $b\hat{g}f$, $c\hat{g}f$, $d\hat{g}f$, $a\hat{g}e$, $b\hat{g}e$, $c\hat{g}e$, $d\hat{g}e$

PART C : MATHEMATICS

Section - 1 :

(Only One option correct Type)

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)\,dx$ lies in the interval

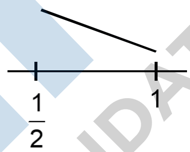
(A) $(2e - 1, 2e)$ (B) $(e - 1, 2e - 1)$ (C) $\left(\frac{e-1}{2}, e-1\right)$ (D*) $\left(0, \frac{e-1}{2}\right)$

Sol. Given, $f'(x) < 2f(x)$

$\therefore f'(x) - 2f(x) < 0 \ \forall \ x \in \left[\frac{1}{2}, 1\right]$

$\therefore \frac{d}{dx}\left(f(x)\cdot e^{-2x}\right) < 0$

Hence, $f(x)\,e^{-2x}$ is a decreasing function in $\left[\frac{1}{2}, 1\right]$.



$f(x)\,e^{-2x} \le f\left(\frac{1}{2}\right) \cdot e^{-1}$.

$f(x)\,e^{-2x} \le 1 \times \frac{1}{e}$

$0 \le f(x) \le e^{2x-1}$

Hence, $0 < \int_{1/2}^{1} f(x)\,dx \le \int_{1/2}^{1} e^{2x-1}\,dx$

$0 < I \le \frac{e-1}{2} \Rightarrow$ (D)

42. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Then the equation of the curve is

(A*) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Ans. **[A]**

Sol. Given, $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$ ......(1)

Put $y = mx$

$\therefore \dfrac{dy}{dx} = m + x\dfrac{dm}{dx}$ ......(2)

From (1) and (2), we get $\dfrac{dm}{\sec m} = \dfrac{dx}{x}$

$\therefore \int \cos m \, dx = \ell n\, x + \lambda \Rightarrow \sin m = \ell n\, x + \lambda.$

Put $\left(1, \dfrac{\pi}{2}\right)$, we get $\lambda = \dfrac{1}{2}$

$\therefore \sin\left(\dfrac{y}{x}\right) = \ell n\, x + \dfrac{1}{2}$. **Ans.**

43. Perpendiculars are drawn from points on the line $\dfrac{x+2}{2} = \dfrac{y+1}{-1} = \dfrac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line

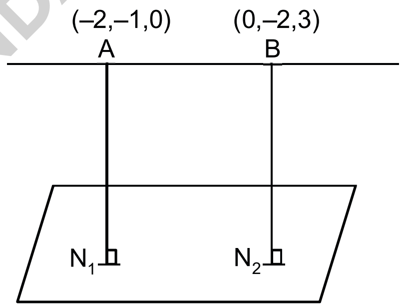
(A) $\dfrac{x}{5} = \dfrac{y-1}{8} = \dfrac{z-2}{-13}$ (B) $\dfrac{x}{2} = \dfrac{y-1}{3} = \dfrac{z-2}{-5}$

(C) $\dfrac{x}{4} = \dfrac{y-1}{3} = \dfrac{z-2}{-7}$ (D*) $\dfrac{x}{2} = \dfrac{y-1}{-7} = \dfrac{z-2}{5}$

Ans. **[D]**

Sol. $L : \dfrac{x+2}{2} = \dfrac{y+1}{-1} = \dfrac{z}{3}$ ....(1)

P : x + y + z = 6 ....(2)



Take 2 points A (– 2, – 1, 0) and B (0, – 2, 3) on line L.

$\therefore$ N_1 is (0, 1, 2)

[Foot of perpendicular of point (A) on plane P.]

and N_2 is $\left(\dfrac{2}{3}, \dfrac{-4}{3}, \dfrac{11}{3}\right)$

[Foot of perpendicular of point (B) on plane P.]

Hence, equation of line joining N_1 and N_2 is $\dfrac{x}{\frac{2}{3}} = \dfrac{y-1}{\frac{-7}{3}} = \dfrac{z-2}{\frac{5}{3}}$ or $\dfrac{x}{2} = \dfrac{y-1}{-7} = \dfrac{z-2}{5}$. **Ans.**

44. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

(A) 5 (B) 20 (C) 10 (D) 30

Sol. **[C]**

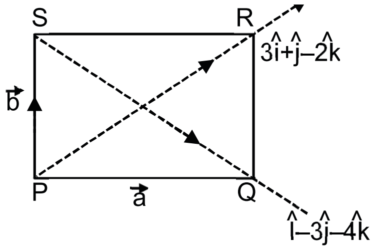
Given, $\vec{a} + \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} - \vec{a} = \hat{i} - 3\hat{j} - 4\hat{k}$

$\therefore\ \vec{a} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$

and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Volume = $\left[\vec{a}\ \vec{b}\ \overrightarrow{PT}\right]$ = 10. **Ans.**



45. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

(A) $\frac{23}{25}$ (B*) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Ans. **[B]**

Sol. $\sum_{k=1}^{n}2k = n(n+1)$

$\therefore \cot^{-1}[1 + n(n+1)] = \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right) = \tan^{-1}(n+1) - \tan^{-1}n$

$\therefore \sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right) = \cot\left[\sum_{n=1}^{23}\tan^{-1}(n+1)-\tan^{-1}n\right]$ = cot $[\tan^{-1}24 - \tan^{-1}1]$

$= \cot\left(\tan^{-1}\frac{23}{25}\right) = \frac{25}{23}$. **Ans.**

46. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then

(A*) a + b – c > 0 (B) a – b + c < 0 (C) a – b + c > 0 (D) a + b – c < 0

Ans. **[A]**

Sol. Given, a > b > c > 0

and ax + by + c = 0

bx + ay + c = 0

Using cramer rule $\frac{x}{bc-ac} = \frac{y}{bc-ac} = \frac{1}{a^2-b^2}$

$\therefore x = \frac{-c}{a+b}$; $y = \frac{-c}{a+b}$

$\therefore$ distance from (1, 1) less than $2\sqrt{2}$.

$\therefore \sqrt{\left(1+\frac{c}{a+b}\right)^2+\left(1+\frac{c}{a+b}\right)^2} \le 2\sqrt{2} \Rightarrow \frac{a+b+c}{a+b} < 2$

$\therefore \frac{c-a-b}{a+b} < 0 \Rightarrow c - a - b < 0 \Rightarrow a + b - c > 0$. **Ans.**

47. Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha|$ =

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C*) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

Ans. [C]

Sol. α lie on $(x - x_0)^2 + (y - y_0)^2 = r^2$

$\therefore\ a = z_0 + re^{i\theta}$ ....(1)

Similarly, $\frac{1}{\alpha}$ lie on $(x - x_0)^2 + (y - y_0)^2 = 4r^2$

$\therefore\ \frac{1}{\alpha} = z_0 + 2re^{i\theta}$ ....(2)

From (2) – (1)

$\therefore\ \frac{1}{\alpha} - \alpha = e^{i\theta} \Rightarrow \quad r^2 = \left(\frac{1}{\bar{\alpha}} - \alpha\right)\left(\frac{1}{\alpha} - \bar{\alpha}\right) = \frac{1}{|\alpha|^2} + |\alpha|^2 = 2|z_0|^2$ ....(3)

From (1) and (2) again.

$$\frac{\alpha - z_0}{\frac{1}{\bar{\alpha}} - z_0} = \frac{1}{2}$$

$\therefore\ z_0 = 2\alpha - \frac{1}{\alpha} \Rightarrow |z_0|^2 = \left(2\alpha - \frac{1}{\bar{\alpha}}\right)\left(2\bar{\alpha} - \frac{1}{\alpha}\right)$

$2|z_0|^2 = 8|\alpha|^2 - 8 + \frac{2}{|\alpha|^2}$ ......(4)

From (3) and (iv)

$7|\alpha|^4 - 8|\alpha|^2 + 1 = 0$

$\therefore\quad |\alpha|^2 = \frac{1}{7} + 1$

$\therefore\quad |\alpha| = \frac{1}{\sqrt{7}}$. **Ans.**

48. The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is

(A) 6 (B) 4 (C*) 2 (D) 0

Ans. [C]

Sol. $f(x) = x^2 - x \sin x - \cos x$

f (x) is an even function

and $f(0) = -1$

and $f'(x) = x(2 - \cos x) > 0$ for $x \in (0, \infty)$

$\therefore$ f (x) is an increasing function

$\therefore$ f (x) must have exactly one real root in $(0, \infty)$

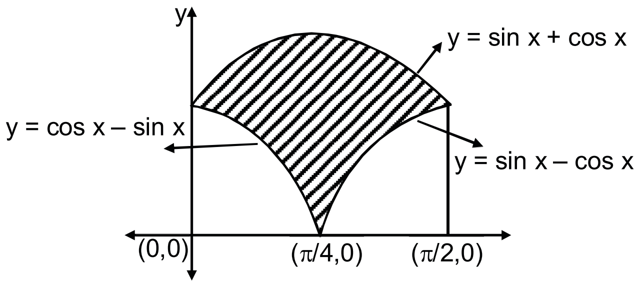
$\Rightarrow$ two real roots.

49. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $4(\sqrt{2} - 1)$ (B*) $2\sqrt{2}(\sqrt{2} - 1)$ (C) $2(\sqrt{2} + 1)$ (D) $2\sqrt{2}(\sqrt{2} + 1)$

Ans. [B]

Sol. $\therefore$ The required area = $\int_0^{\pi/2} (\sin x + \cos x)dx$



$= (-\cos x + \sin x)_0^{\pi/2} - 2(\sin x + \cos x)_0^{\pi/2}$

$= (0 + 1) - (-1 + 0) - 2\left(\sqrt{2} - 1\right)$

$= 2 - 2\sqrt{2} + 2$

$= 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2} - 1)$. **Ans.**

50. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by atleast one of them is

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Sol. [A]

$P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$

$P(C) = \frac{1}{4}$, $P(s) = \frac{1}{8}$

Now, P (problems is solved correctly by atleast one of them)

= 1 – P (none of them solved the problem correctly)

$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})\, P(\bar{D}) = 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$. **Ans.**

Section - 2 :

(One or more options correct Type)

This section contains **5 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

51. For 3 × 3 matrices M and N, which of the following statement(s) is(are) **not** correct?

(A) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric.

(B) MN – NM is skew symmetric for all symmetric matrices M and N.

(C*) MN is symmetric for all symmetric matrices M and N.

(D*) (adj M) (adj N) = adj (MN) for all invertible matrices M and N.

Ans. **[CD]**

Sol.

(A) Let $P = N^TMN$

$\Rightarrow$ $P^T = (N^TMN)^T = N^T M^T N$

$\Rightarrow$ $P^T = P$, if $M^T = M$

or $P^T = -P$, if $M^T = M$

or $P^T = -P$, if $M^T = -M$

$\Rightarrow$ True

(B) Let $Q = MN - NM$

$\Rightarrow$ $QT = (MN - NM)^T$

$= (MN)^T - (NM)^T$

$= N^T M^T - M^T N^T$

$= NM - MN = -Q$

$\Rightarrow$ True

(C) Let $R = MN$

$\Rightarrow$ $R^T = (MN)^T = N^T MT$

$= NM$

$\Rightarrow$ False

(D) As, adj (MN) = (adj. N)(adj. M)

$\Rightarrow$ False.

52. A line ℓ passing through the origin is perpendicular to the lines

$\ell_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$

$\ell_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_1 is (are)

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) $(-1, -1, 0)$ (C) $(1, 1, 1)$ (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Sol. BD

$\ell_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = \lambda$

$\ell_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$

$\therefore$ line ℓ is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$\Rightarrow$ $\ell : \frac{x-0}{2} = \frac{y-0}{3} = \frac{z-0}{-2} = \mu$

Any point on ℓ_1 is $(\lambda + 3, 2\lambda - 1, 2\lambda + 4)$

and Any point on ℓ is $(-2\mu, 3\mu, -2\mu)$

$\therefore$ on solving ℓ_1 and ℓ,

we get (2, – 3, 2)

Now, any point on ℓ_2 is

$(2s + 3, 2s + 3, s + 2)$

No, $(2s + 1)^2 + (2s + 6)^2 + s^2 = 17$

$\Rightarrow \quad 9s^2 + 28s + 20 = 0 \begin{cases} -2 \\ \frac{-10}{9} \end{cases}$

$\therefore$ for s = – 2, we get (–1, – 1, 0) and for $s = \frac{-10}{9}$, we get $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$.

53. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(A*) 24 (B) 32 (C*) 45 (D) 60

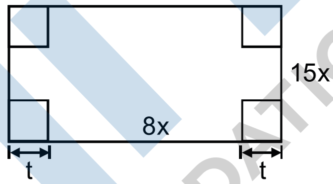
Ans. **[AC]**

Sol. Let sides of rectangular sheet be 8x and 15x where (x is fixed because perimeter of rectangular sheet is constant.)

Now, $V(t) = (8x - 2t)(15x - 2t)\,t = (4t^3 - 46xt^2 + 120x^2t)$

$V'(t) = 12t^2 - 92xt + 120x^2$

$V'(t) = 0$



$\Rightarrow \quad 3t^2 - 23xt + 30x^2 = 0$

$\Rightarrow \quad (3t - 5x)(t - 6x) = 0$

$\therefore \quad \frac{t}{x} = \frac{5}{3}$ or $t = 6x$ (reject)

$\Rightarrow \quad t = \frac{5}{3}x$

Now, $4t^2 = 100 \Rightarrow t^2 = 25 \Rightarrow t = 5 \Rightarrow x = 3$

$\therefore$ sides are $8x = 8 \times 3 = 24$

and $15x = 15 \times 3 = 45$

54. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s)

(A*) 1056 (B) 1088 (C) 1120 (D*) 1332

Ans. **[AD]**

Sol. $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$

$= -(1)^2 - (2)^2 + (3)^2 + (4)^2 - (5)^2 - (6)^2 + (7)^2 + (8)^2 - (9)^2 - (10)^2 + (11)^2 + (12)^2 +$

$\ldots\ldots\ldots + (4n-1)^2 + (4n)^2$

$= (3^2 - 1^2) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2) + (11^2 - 9^2) + (12^2 - 10^2) +$

$\ldots\ldots\ldots + \left((4n-1)^2 - (4n-3)^2\right) + \left((4n)^2 - (4n-2)^2\right)$

= 2 [4 + 12 + 20 + ……. upto n terms] + 2 [6 + 14 + 22 + ……. upto n terms]

= 2 (8 + (n – 1) 8) + n (12 + (n – 1) 8)

$S_n = 4n\ (4n + 1)$

$\therefore\ S_8 = 32 \times 33 = 1056$

and $S_9 = 36 \times 37 = 1332$.

55. Let f(x) = x sin πx, x > 0. Then for all natural numbers n, f '(x) vanishes at

(A) A unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(B) a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(C) a unique point in the interval (n, n + 1)

(D) two points in the interval (n, n + 1)

Sol. **[BC]**

$f(x) = x \sin \pi x\ ; \quad x > 0$

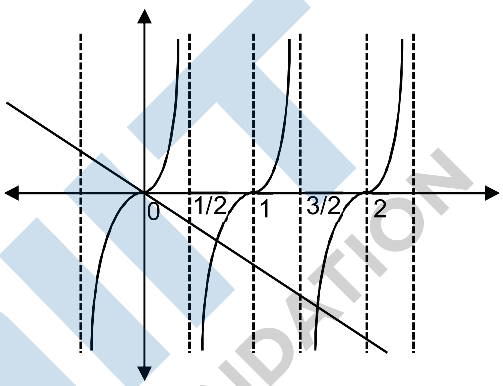
$f'(x) = \sin \pi x + \pi x \cos \pi x$

$f'(x) = 0 \quad \Rightarrow \quad \tan(\pi x) = -(\pi x)$

similar like, $\tan \theta = -\theta$

Graph are intersecting in $\left(\frac{1}{2}, 1\right)$ and $\left(\frac{3}{2}, 2\right)$

option (B) and (C) are satisfying.



Section - 3 :

(Integer value correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

56. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 =

Ans. **[5]**

Sol. The smaller number removed = k

The next number removed = k + 1

$\therefore \quad 1 + 2 + 3 + \ldots\ldots\ldots + n = (k) + (k + 1) + 1224$

$\frac{n(n+1)}{2} = 2k + 1225$

$\Rightarrow \quad n^2 + n = 4k + 2450$

$\Rightarrow \quad n^2 + n - 2450 = 4k$

$\Rightarrow \quad (n + 50)(n - 49) = 4k$

Here $1 < k < n$ and either of (n + 50) or (n – 49)

must be a multiple of '4' as

because if n is odd then (n – 49) is even

and if n is even then (n + 50) is even

so, for n = 50, k = 25

but for n = 53 ; k = 103 $\Rightarrow$ k > n (not allowed)

rest values of n are not allowed

$\therefore$ k = 25 and k – 20 = 5 **Ans.**

57. Consider the set of eight vectors $V = \left\{ a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\} \right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p, is

Ans. **[5]**

Sol. 8 vectors of given type are as follows

(1, 1, 1) $\longleftrightarrow$ (–1, – 1, – 1)

(1, 1, – 1) $\longleftrightarrow$ (–1, – 1, 1)

(1, – 1, 1) $\longleftrightarrow$ (–1, 1, – 1)

(– 1, 1, 1) $\longleftrightarrow$ (1, – 1, – 1)

The given pairs are collinear (anti parallel) any three pairs will be selected from the '4' available pairs and from each pair any one vector will be selected.

$\therefore$ $^4C_3 \times {^2C_1} \times {^2C_1} \times {^2C_1} = {^4C_3} \times 2^3 = 2^5 = 32$

i.e. $2^p = 32$

$p = 5$

58. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$

Ans. 9

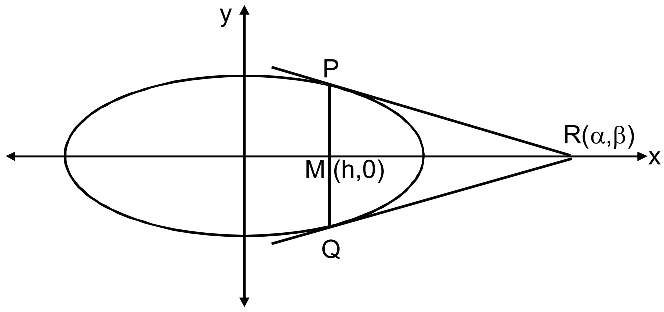
Sol. $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Equation of PQ : x – h = 0

If R (α, β) then chord of contact of R is PQ i.e. C.O.C. = T = 0

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{3} = 1 \quad .....(1)$

and also $\frac{x}{h} = 1 \quad .....(2)$



Comparing (1) and (2)

$$\frac{\frac{\alpha}{4}}{\frac{1}{h}} = \frac{\frac{8}{3}}{0} = \frac{1}{1}$$

$\Rightarrow \quad \alpha = \frac{4}{h};\ \beta = 0 \quad \Rightarrow \quad$ Point R is $\left(\frac{4}{h}, 0\right)$

coordinate of P and Q are respectively $P\left(h_1 + \sqrt{\left(1+\frac{h^2}{4}\right)^3}\right)$ and $Q\left(h_1 - \sqrt{\left(1-\frac{h^2}{4}\right)^3}\right)$

area (ΔPQR) = $\frac{1}{2}$(PQ) (MR) $= \frac{1}{2} \times 2\sqrt{\left(1-\frac{h^2}{4}\right)^3} \times \left(\frac{4}{h} - h\right)$

$\Delta(h) = \frac{(4-h^2)^{3/2}\sqrt{3}}{2h}$

Since $\Delta(h)$ is decreasing

$\therefore \quad \Delta_1 = \Delta(h)_{max.} = \Delta\left(\frac{1}{2}\right) = \frac{45}{8}\sqrt{5}$ and $\Delta_2 = \Delta(h)_{min.} = \Delta(1) = \frac{9}{2}$

Hence, $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = 9$. **Ans.**

59. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5: 10 : 14. Then n =

Ans. **[6]**

Sol. Let the three terms are T_r, T_{r+1}, T_{r+2} and three coefficients are ${}^{n+5}C_{r-1}$, ${}^{n+5}C_r$, ${}^{n+5}C_{r+1}$

Note: $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

Here, $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = \frac{10}{5} \Rightarrow \frac{n+5-r+1}{r} = 2 \Rightarrow n + 5 - r = 2r \Rightarrow n + 6 = 3r$.......(1)

and $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{14}{10}$

$\Rightarrow \quad \frac{n+5-r}{r+1} = \frac{7}{5}$

$\Rightarrow \quad 5n + 25 - 5r = 7r + 7 \quad \Rightarrow \quad 5n + 18 = 12r \qquad ....(2)$

solving (1) and (2),

$5n + 18 = 4 \times (n + 6)$

$\Rightarrow \quad n = 6.$

60. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$.

All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurence of } E_1}{\text{Probability of occurence of } E_3} =$

Ans. **[6]**

Sol. Let the probability of occurrence of event E_1, E_2 and E_3 are a, b and c.

$\therefore\ P(E_1) = a,\ P(E_2) = b$ and $P(E_3) = c$

Given,

$P\ (\text{only } E_1 \text{ occur}) = P\ (E_1\bar{E}_2\bar{E}_3) = a\ (1 - b)\ (1 - c)$ ……. (1)

$P\ (\text{only } E_2 \text{ occur}) = P\ (E_1\bar{E}_2\bar{E}_3) = (1 - a)\ b\ (1 - c)$ ……. (2)

$P\ (\text{only } E_3 \text{ occur}) = P\ (E_1\bar{E}_2\bar{E}_3) = (1 - a)\ (1 - b)\ c$ ……. (3)

$P\ (\text{none of event occur}) = p = P\ (E_1\bar{E}_2\bar{E}_3)$

$\Rightarrow P = (1 - a)\ (1 - b)\ (1 - c)$ …….(4)

Given, $(\alpha - 2\beta)\, p = \alpha\beta$

$\therefore\ \therefore \left(\dfrac{1}{\beta} - \dfrac{2}{\alpha}\right)p = 1$

Put value of p, α, β from equation (1), (2), (3) and (4)

we get $\dfrac{1}{b} = \dfrac{2}{a}$ …….(5)

Given, $(\beta - 3\gamma)\, p = 2\beta\gamma \Rightarrow \left(\dfrac{1}{\gamma} - \dfrac{3}{2\beta}\right)p = 2$

Put value of γ, β and p, we get

$\dfrac{1}{c} = \dfrac{3}{b}$ …….(6)

From (5) and (6), we get $\dfrac{a}{c} = 6$.**Ans.**